4 Model solution

In this section, we present the model solution. Given our functional form assumptions, our model has a homogeneity property. We show that model can be restated in terms of a single state variable, $x_t \equiv X_t/K_t$, without loss of generality. Using this single state variable, which represents book leverage, greatly simplifies analysis. We characterize a solution to the model in which the value function for equity holders is equal to $P(K_t, X_t) = p(x_t)K_t$ for a function $p(x_t)$. Specifically, we show that the equity holders' optimal action at each t is determined by the value of x_t and three endogenous cutoffs: (i) a payout boundary $\underline{x}$, such that the firm issues debt to pay a dividend when $x_t < \underline{x}$; (ii) an equity-issuance boundary $\widehat{x}$, such that the firm issues equity to reduce leverage when $x_t > \widehat{x}$; and (iii) a default boundary $\overline{x}$, such that the firm defaults the first time that a capital shock brings x_t above $\overline{x}$. In the model solution, $\underline{x} < \widehat{x} < \overline{x}$. Whenever $x_t \in [\underline{x}, \widehat{x}]$, the firm relies on debt financing. It issues (pays down) debt when the free cash flow is less than (greater than) interest expenses. The firm sets its secured-debt policy $s_t = s(x_t)$ and investment $I_t = i(x_t)K_t$ according to functions of the state variable x_t. Finally, the endogenous credit spreads $\eta_t^S = \eta^S(x_t)$ and $\eta_t^U = \eta^U(x_t)$ are functions of x_t, taking into account the firm's optimal policies. Lenders optimally accept liability-management offers when x_t exceeds an endogenous cutoff x_α.

Readers less interested in the technical details may skip to Section 5, which uses this characterization of the optimal firm strategy to present the main results of the paper.

4.1 Payout region

The endogenous boundaries $\underline{x}, \widehat{x}, \overline{x}$ in our conjectured solution partition the state space $[0, \infty)$ of the state variable x_t into four regions. We now characterize each region, starting with the payout region: $x_t < \underline{x}$.

When $x_t = X_t/K_t$ is below the endogenous payout boundary $\underline{x}$, the firm makes a lump-sum payment $(\underline{x} - x_t)K_t$ to shareholders. The lump-sum payment is financed with debt, bringing x_t to $\underline{x}$. The equity value function p must then satisfy the following value-continuity condition for $x < \underline{x}$:

$$p(x) = p(\underline{x}) + \underline{x} - x\,, \quad \text{for} \quad x < \underline{x}\,. \tag{21}$$

Since (21) holds for x close to $\underline{x}$, we obtain the following smooth-pasting condition for $\underline{x}$:

$$p'(\underline{x}) = -1\,, \tag{22}$$

by taking the limit $x \to \underline{x}$. At $x = \underline{x}$, equity holders are indifferent between reducing debt by one dollar and distributing this dollar to shareholders. Since the payout boundary $\underline{x}$ is an optimal choice, we also have the following super-contact condition (see, e.g., Dumas, 1991):

$$p''(\underline{x}) = 0\,. \tag{23}$$

4.2 Equity issuance region

We next characterize the endogenous equity-issuance region: $\widehat{x} \le x \le \overline{x}$. If leverage x_t enters this region, then the firm issues equity, choosing the net issuance proceeds M_t. Define $m_t \equiv M_t/X_t$. We conjecture an equity value function $p(x)$ that is continuous before and after issuance. That implies the following value-matching condition holds for $x_t \in [\widehat{x}, \overline{x}]$:

$$p(x_t) = \max_{m>0} \Big[p(x_t - m) - [h_0 + (1 + h_1)m]\Big]. \tag{24}$$

This value-matching condition simply requires that new and old shareholders break even. In other words, the sum of the equity-issuance costs $h_0 + h_1 m_t$ and the dollars injected m_t must equal the value of the equity that shareholders receive, $p(x_t - m_t) - p(x_t)$.

We can define $\widetilde{x} \equiv x_t - m$ and plug this in above to rewrite the maximization as

$$\max_{\widetilde{x}} p(\widetilde{x}) - [h_0 + (1 + h_1)(x_t - \widetilde{x})] \,. \tag{25}$$

We see that the maximization (25) is independent of the value of x_t. This implies that for any x_t in the equity-issuance region $[\widehat{x}, \overline{x}]$, the firm chooses the same post-issuance target leverage $\widetilde{x}$. This equity-issuance target leverage is characterized by the argmax of (25) over the region $\widetilde{x} \in [\underline{x}, \widehat{x}]$, since the post-issuance leverage will be below the issuance boundary $\widehat{x}$. Note that the equity-issuance target leverage $\widetilde{x}$ is higher than the debt-financed target leverage $\underline{x}$ because each dollar of equity issued has a marginal cost.

Finally, we determine the firm's optimal equity-issuance boundary $\widehat{x}$. Since the target $\widetilde{x}$ does not depend on x, (24) implies that for any $x \in [\widehat{x}, \overline{x}]$,

$$p(x) = p(\widetilde{x}) - [h_0 + (1 + h_1)(x - \widetilde{x})]. \tag{26}$$

This should hold at the boundary $\widehat{x}$, so

$$p(\widehat{x}) = p(\widetilde{x}) - [h_0 + (1 + h_1)(\widehat{x} - \widetilde{x})]. \tag{27}$$

Combining these, for any $x \in [\widehat{x}, \overline{x}]$,

$$p(x) = p(\widehat{x}) - (1 + h_1)(x - \widehat{x}). \tag{28}$$

Since $p(x)$ is continuous and differentiable at the endogenous equity-issuance boundary $\widehat{x}$, we can find the equity-issuance boundary $\widehat{x}$ by imposing the following value-matching and smoothing-pasting conditions:

$$\lim_{x \uparrow \widehat{x}} p(x) = p(\widetilde{x}) - [h_0 + (1 + h_1)(\widehat{x} - \widetilde{x})] \tag{29}$$

$$\lim_{x \uparrow \widehat{x}} p'(x) = -(1 + h_1). \tag{30}$$

4.3 Default Region

Next, we characterize the default region $x > \overline{x}$. Intuitively, there is no point in equity holders voluntarily defaulting while $P(K, X) = p(x)K$ is strictly positive. Likewise, default is strictly preferred if $p(x)$ is strictly negative. It follows that equity holders voluntarily default the first time that $p(x) \leq 0$. If $p(x)$ would be negative, then equity holders default, so limited liability implies that:

$$p(x) = 0\,, \text{ when } \; x \geq \overline{x}. \tag{31}$$

Substituting $p(\overline{x}) = 0$ into the linear equity valuation equation (28), we obtain the following relation between the default boundary $\overline{x}$ and the equity-issuance boundary $\widehat{x}$:

$$\overline{x} - \widehat{x} = \frac{p(\widehat{x})}{1 + h_1}\,. \tag{32}$$

Note that since $\overline{x} > \widehat{x}$, a voluntary default cannot occur unless a capital shock arrives: a Brownian shock would push the firm into the equity-issuance region before reaching the default region and the firm would issue equity to lower leverage. In the absence of a restructuring, the firm thus defaults if and only if there is a capital shock and Z is sufficiently low that

$$x_t = \frac{1}{Z}x_{t-} > \overline{x}. \tag{33}$$

Rearranging, we can define a default threshold $Z_*(x) \equiv x/\overline{x}$ in capital-shock space. In the absence of a restructuring, the firm defaults the first time that $Z < Z_*(x_t)$. Finally, if a restructuring is accepted, the debt level falls from X_t to $X_t(1 - s_t\zeta(1-\varepsilon))$. The above argument implies the firm then defaults if and only if $Z < Z_*(x(1-s\zeta(1-\varepsilon))) \equiv Z_*^{res}(x,s)$.

4.4 Earnings retention and debt-financing region

When $\underline{x} < x_t < \widehat{x}$, equity holders do not want to pay out cash ($x_t > \underline{x}$) or issue equity ($x_t < \widehat{x}$). Intuitively, in this region, leverage is too high to justify issuing a debt-financed dividend. However, the costs of leverage deviating from the target level are too small in this region to justify the equity-issuance costs needed to reach target leverage. In this region, the firm's leverage thus evolves stochastically, deviating from the target leverage. The firm pays down or grows its debt outstanding depending on whether its free cash flow is higher or lower than its interest expense.

Formally, combining equations (1) and (12) and noting there are no payouts and no equity issuance in this region, we can apply Ito's lemma for semimartingales[16] to derive the evolution of $x_t = X_t/K_t$:

[16] See, for example, Lemma 3 of Appendix H of Duffie (2010).

$$
\begin{aligned}
dx_t = d\frac{X_t}{K_t} &= \frac{X_t}{K_t}\left(\frac{dX_t}{X_t} - \frac{dK_t}{K_t} + \Big(\frac{dK_t}{K_t}\Big)^2\right)\mathbf{1}_{d\mathcal{J}_t=0} + \Big(\frac{X_t}{K_t} - \frac{X_{t-}}{K_{t-}}\Big)d\mathcal{J}_t \\
&= x_t\left(\frac{-[\theta K_t - I_t - (1-\tau)C_t]dt}{X_t} - \Big[\Big(\psi\left(\frac{I_{t-}}{K_{t-}}\right) - \delta\Big)dt + \sigma d\mathcal{B}_t\Big] + \sigma^2 dt\right)\mathbf{1}_{d\mathcal{J}_t=0} \\
&+ \Big(\frac{X_{t-}(1-\mathbf{1}_t^R\zeta(1-\varepsilon)s_{t-})}{K_{t-}Z} - \frac{X_{t-}}{K_{t-}}\Big)d\mathcal{J}_t \\
&= \Big(-\theta + i(x_t) + (1-\tau)c_t + x_t\big[-\psi(i(x_t)) + \delta + \sigma^2\big]\Big)dt - \sigma x_t dB_t \\
&\quad + \Big(\frac{x_{t-}(1-\mathbf{1}_t^R\zeta(1-\varepsilon)s_{t-})}{Z} - x_{t-}\Big)d\mathcal{J}_t, \qquad (34)
\end{aligned}
$$

where $c_t \equiv C_t/K_t = x_t[r + s_t\eta_t^S + (1-s_t)\eta_t^U]$.

Given these dynamics, in Appendix A.3, we show that the Hamilton-Jacobi-Bellman (HJB) equation for the value function $P(K_t, X_t)$ of equity holders implies the following HJB for $p(x_t)$ over the region $x \in (\underline{x}, \widehat{x})$:

$$
\begin{aligned}
\gamma p(x) = &\max_{i,s\in[0,\min\{1,\frac{\pi}{x}\}]}\Big(-\theta + i + (1-\tau)c(x,s,Z_*(x))\Big)p'(x) + \frac{1}{2}\sigma^2x^2p''(x) \qquad (35)\\
&+ \Big(\psi(i) - \delta\Big)\Big(p(x) - xp'(x)\Big) + \phi\big(sx\big)^{\nu}\left[(\pi - \rho - x)^+ - p(x)\right] \\
&+ \lambda\left[(1-\alpha\mathbf{1}^R(x))\int_{Z_*(x)}^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) + \alpha\mathbf{1}^R(x)\int_{Z_*^{res}(x,s)}^1 Zp\Big(x\frac{1-s\zeta(1-\varepsilon)}{Z}\Big)dF(Z) - p(x)\right].
\end{aligned}
$$

We now explain this equation. Recall that $Z^{res}(x,s), Z_*(x)$ are the necessary shock sizes to induce a default with or without a restructuring, respectively. In Appendix A, we derive a function $c(x,s,Z_*(x))$ such that lenders with rational expectation will charge a coupon $C_t = K_tc(x_t,s_t,Z_*(x_t))$ to break even. This incorporates the role of secured debt in determining the credit spread. The first three terms of (35) capture the sensitivity of equity value to continuous stochastic fluctuations in leverage, given the endogenous secured-debt ratio, investment spending, and credit spreads. The fourth term captures the impact of a secured-lender takeover.

The final line of (35) captures the impact of capital shocks. We derive a cutoff x_α and a function $\mathbf{1}^R(x) = \mathbf{1}(x > x_\alpha)$ such that secured lenders optimally accept a restructuring offer if and only if $x_t > x_\alpha$ (i.e., $\mathbf{1}^R(x_t) = 1$). The probability of a restructuring after a shock is thus $\alpha\mathbf{1}^R(x)$. The first term on this line captures how, in the absence of a restructuring, a capital shock lowers K_t and leaves X_t fixed. The final term captures how a capital shock followed by a restructuring lowers both K_t and X_t.

In this debt-financing region, equity holders choose investment spending $i = i(x_t)$ and secured debt $s_t = s(x_t)$ to maximize the right side of the HJB. Taking a derivative, we can show analytically that the optimal investment level is

$$i_*(x) = \frac{1}{\xi}\left(1 - \frac{p'(x)}{xp'(x) - p(x)}\right). \tag{36}$$

The optimal investment level thus trades off the cost ξ associated with increasing the capital stock with the increase in value from lowering leverage with another unit of capital.

4.5 Numerical solution

The solution method for our jump-diffusion model is different from pure-diffusion models, which only require local information around x. Moreover, the circularity between creditor choices (credit spreads and restructuring acceptance) and equity holder choices introduces complication. Our numerical algorithm accounts for this with an iterative approach. We guess a function $p_i(x)$ with associated boundaries $\underline{x}, \widehat{x}, \overline{x}$ and calculate credit spreads and restructuring acceptance decisions. We then use the HJB (35) and other conditions described above to update to a new guess $p_{i+1}(x)$, assuming equity holders' strategies, creditor behavior and post-jump-shock values derived from $p_i(x)$. We repeat until this process converges. We provide details on our numerical method in Appendix A.4.

5 Results

This section presents our main results. In Section 5.1, we provide intuition for how the firm optimizes the path of its leverage x_t. In Section 5.2, we characterize the optimal secured-debt ratio $s_*(x_t)$. In Section 5.3, we conduct comparative statics with respect to the parameter α to show our main result: more frequent liability management leads to higher secured-debt-credit spreads and lower secured-debt use, but also prevents liquidations and increases both investment and ex-ante firm value. Finally, Section 5.4 shows that our model matches empirical evidence.

5.1 The optimal leverage ratio

First, we build intuition for our model by studying the leverage dynamics implied by our model solution. We solve our model numerically assuming the parameters given in Table 1. Recall that whenever $x_t < \underline{x}$, the firm immediately issues debt and pays a dividend to bring leverage up to $\underline{x}$. Likewise, whenever $x_t > \widehat{x}$, the firm immediately issues equity to bring leverage down to $\widetilde{x}$. The firm's leverage thus remains in the range $[\underline{x}, \widehat{x}]$, almost surely, prior to default (which occurs if a jump shock brings x from $[\underline{x}, \widehat{x}]$ to a value above $\overline{x}$).

Figure 1 displays the model solution in the range $x_t \in [\underline{x}, \overline{x}]$. As expected, panel A shows that the ex-post enterprise value declines in x in this range. By definition, $\underline{x}$ is the point at which equity holders are indifferent between keeping leverage fixed or issuing another dollar of debt to pay a dividend. For $x > \underline{x}$, it follows that $p'(x) < -1$ and thus the ex-post enterprise value $v(x) = p(x) + x$ declines in x. In this sense, the firm's leverage is typically higher than its debt-financed target leverage $\underline{x}$. Once leverage rises to $\widehat{x}$, the firm incurs the equity-issuance cost to issue equity and lower leverage. Since equity issuance has a marginal cost per dollar of equity issued, the firm's equity-financed target leverage $\widetilde{x}$ is higher than its debt-financed target $\underline{x}$. For $x > \widehat{x}$, the declining firm value simply reflects the higher equity-issuance costs needed to bring down leverage.

Interestingly, panel B shows that the enterprise value is concave in x for low x_t and convex in x for high values of x_t. Because of this, panel D shows that investment first falls as leverage rises (debt overhang) for low leverage levels, then increases with leverage (risk-shifting). Panel C shows the obvious result that market leverage $x/v(x)$ increases as book leverage rises.

What motivates the firm's choice of the endogenous thresholds $\underline{x}, \widehat{x}$ determining leverage dynamics? Figure 2 shows that the answer is a standard tradeoff theory. As we increase the tax rate τ, the simulated average leverage rises. As we increase the value of the firm in default (e.g., shrink the deadweight losses), the simulated average leverage also rises.

5.2 The optimal secured ratio

Next, we illustrate the choice of secured debt in our model. The benefit of secured debt is that it allows firms to lower their cost of credit. This lower cost of credit arises because secured lenders are senior to existing priority unsecured claims, such as wages. Issuing secured debt essentially allows the firm to transfer value from priority claim holders to secured claim holders. The downside of secured debt is that secured creditors push for early default to ensure full recovery (Section 3.4.3). This can lead to an early default that lowers firm value.

We provide intuition for this tradeoff driving the secured-debt choice using comparative statics. In Figure 3, we vary the parameter ρ that captures priority unsecured claims. For each fixed value of ρ, we solve the model. The right panel of Figure 3 shows that as ρ increases, the firm optimally chooses a higher secured-debt ratio: the simulated average secured-debt ratio rises. This is explained by the same intuition described above. As ρ increases, the recovery value available to unsecured claims declines. Secured debt then becomes more valuable because it lowers the cost of credit by skipping ahead of priority claims. Next, we vary the parameter ϕ driving the probability of a forced default. As ϕ increases, secured lenders are more likely to push for early default to ensure full recovery. Default imposes a deadweight loss because the recovery value is lower than the firm value. Secured lenders do not care about this deadweight loss since they still get full recovery in a forced default. However, equity holders internalize this deadweight loss because it increases the cost of unsecured credit and lowers the expected value of future dividends. Because of this, the left panel of Figure 3 shows that the simulated average secured-debt ratio falls as ϕ increases.

Figure 1 provides further intuition on secured-debt use. Panel F shows that credit spreads rise as x_t rises. This is intuitive. For higher levels of x_t, default becomes more likely. This occurs because the set of default-inducing Z shocks increases. Specifically, default occurs when a sufficiently low realization of Z causes $X_t/K_t = X_t/(ZK_{t-})$ to rise above $\overline{x}$. As $x_{t-} = X_{t-}/K_{t-}$ rises, this becomes more likely. Panel F also shows the obvious result that unsecured credit spreads are higher than secured credit spreads. The gap between secured and unsecured spreads rises as x_t increases due to the higher likelihood of default. This motivates the firm to use more secured debt as x_t rises (panel E).

Finally, to show how the firm's overall financial strategy changes with secured-debt use, we impose an exogenous upper limit $\overline{s}$ on secured debt. We solve our model as before with an additional constraint that $s_t < \overline{s}$. Increasing $\overline{s}$ demonstrates how secured-debt use impacts a firm. In Figure 4, we increase $\overline{s}$ and solve the model at each value. Figure 4 shows that ex-ante firm value $v(0)$ increases as the firm is able to use more secured debt. The firm stops benefiting once $\overline{s}$ rises above the optimal secured-debt ratio, so the constraint doesn't bind. Figure 4 also shows that the increased use of secured debt leads to a higher probability of default due to forced takeovers by secured lenders. Figure 5 shows that the firm uses more leverage as its ability to use secured debt rises. As a result of the higher leverage, both secured and unsecured credit spreads rise. However, Figure 5 shows that at a certain point the weighted credit spread η nonetheless falls as $\overline{s}$ rises. This is the benefit of secured debt — it allows the firm to extract value from workers to lower the cost of credit for a given level of leverage.

5.3 The rise of liability management

In our analysis thus far, we have assumed no liability-management transactions occur ($\alpha = 0$ in Table 1). We now consider the impact of the recent trend toward more frequent liability-management transactions. We increase the parameter α to 0.8 and solve our model. The left panel of Figure 6 shows that there exists a cutoff x_α such that secured lenders optimally accept a liability-management transaction if and only if $x_t > x_\alpha$. The right panel of Figure 6 shows that as the haircut falls (ε rises), secured lenders are more likely to accept an offer (x_α falls).

Next, we study what happens as liability-management transactions become more common. We increase α along a grid of values, solving the model and solving for x_α. Panel B of Figure 7 shows that as restructurings become more common, secured credit spreads increase. Secured lenders anticipate having to either get subordinated or pay a haircut in a future restructuring. As a result, panel C of Figure 7 shows that secured-debt use falls as α rises. This in turn leads to fewer defaults as secured-debt takeovers become less common (panel F).

Surprisingly, Figure 7 shows that ex-ante firm value nonetheless increases as α increases (panel A). The intuition is the following. In a restructuring, value is transferred from secured lenders to equity holders. Secured lenders price this in ex-ante, so it has no impact on firm value. Additionally, restructurings lower debt when they are not followed by default. Ex-ante, lenders also charge a higher spread for this fact. However, this is more than a transfer. Lowering debt in bad states of the world (high leverage) increases enterprise value. Equity holders are not willing to pay for a debt reduction in these states due to a debt-overhang problem: part of the benefit goes to lenders. By allowing equity holders to lower debt for free ex-post, liability-management transactions solve this debt-overhang problem. Thus, while lenders charge more ex-ante, there is nonetheless value created for equity holders ex-ante by the ex-post flexibility. This is why ex-ante value increases as liability-management transactions become more common.

5.4 Matching empirical evidence

Finally, we show that our model produces realistic firm debt policies. This serves as a validation of the model's prediction regarding the trend toward creditor-on-creditor violence.

Using the parameters from Table 1, we solve our model. Table 2 shows the optimal firm leverage is 37%. The optimal secured debt ratio is 33.9%. Benmelech, Kumar, and Rajan

(Forthcoming) show in their Table 2 that the average secured debt share is 33% and the average leverage ratio is 37%. In this sense, our model perfectly replicates observed leverage and secured-debt use.

Benmelech, Kumar, and Rajan (2022) compare credit spreads on secured and unsecured debt issued by the same firm at the same time. They show that the senior secured credit spread is 222 basis points lower than the junior unsecured credit spread (Table 2 column 4). Our Table 2 replicates the same exercise in our model, showing that secured credit spreads are 284 basis points lower than unsecured credit spreads.

Finally, Benmelech, Kumar, and Rajan (Forthcoming) show that firms issue more debt in crises and when they are in distress. Panel E of Figure 1 shows that firms in our model use more secured debt as negative shocks drive their leverage above their target. In this sense, our model replicates this fact.

6 Conclusion

We build a continuous-time capital structure model in which a firm chooses its investment, leverage, secured-debt ratio, payout policy, equity issuance, and default timing. We show that the choice of leverage is determined by a standard tradeoff between the tax benefits of debt and the costs of default. We show that the secured-debt share is chosen by a novel tradeoff between a lower cost of credit, due to the ability to extract value from priority unsecured claims like wages, and a higher probability of default, due to secured-lender incentives to push for early asset sales.

Within this model, we introduce a recent phenomenon: secured lenders have used legal loopholes to extract value from other secured lenders when firms become distressed. We show that this recent trend increases the cost of secured debt and endogenously lowers secured-debt use. However, the lower use of secured debt also leads to fewer defaults, since there is less incentive to push for early asset sales. Moreover, the libaility-management transactions create value ex-ante by allowing the firm to introduce state-contingent debt reduction.

References

Abel, A. B. 2018. Optimal debt and profitability in the trade-off theory. *The Journal of Finance* 73:95–143.

Antill, S. 2022. Do the right firms survive bankruptcy? *Journal of Financial Economics* 144:523–46.

———. 2024. Are bankruptcy professional fees excessively high? *Available at SSRN 3554835* .

Antill, S., and S. R. Grenadier. 2019. Optimal capital structure and bankruptcy choice: Dynamic bargaining versus liquidation. *Journal of Financial Economics* 133:198–224.

Antill, S., and M. Hunter. 2023. Consumer choice and corporate bankruptcy. *Available at SSRN 3879775* .

Auclert, A., and M. Rognlie. 2016. Unique equilibrium in the eaton–gersovitz model of sovereign debt. *Journal of Monetary Economics* 84:134–46.

Ayotte, K. M., and E. R. Morrison. 2009. Creditor control and conflict in chapter 11. *Journal of Legal Analysis* 1:511–51.

Becher, D. A., T. P. Griffin, and G. Nini. 2022. Creditor control of corporate acquisitions. *The Review of Financial Studies* 35:1897–932.

Benmelech, E., N. Kumar, and R. Rajan. 2022. The secured credit premium and the issuance of secured debt. *Journal of Financial Economics* 146:143–71.

———. Forthcoming. The decline of secured debt. *Journal of Finance* .

Bolton, P., H. Chen, and N. Wang. 2011. A unified theory of tobin's q, corporate investment, financing, and risk management. *The journal of Finance* 66:1545–78.

Bolton, P., and D. S. Scharfstein. 1996. Optimal debt structure and the number of creditors. *Journal of political economy* 104:1–25.

Bolton, P., N. Wang, and J. Yang. 2021. Leverage dynamics under costly equity issuance. *NBER Working Paper* 26802.

Bris, A., and I. Welch. 2005. The optimal concentration of creditors. *The Journal of Finance* 60:2193–212.

Broadie, M., M. Chernov, and S. Sundaresan. 2007. Optimal debt and equity values in the presence of chapter 7 and chapter 11. *The journal of Finance* 62:1341–77.

Buccola, V. S. 2023. Sponsor control: A new paradigm for corporate reorganization. *U. Chi. L. Rev.* 90:1–.

Buccola, V. S., and G. Nini. 2022. The loan market response to dropdown and uptier transactions. *Available at SSRN* .

Donaldson, J. R., D. Gromb, and G. Piacentino. 2019. Conflicting priorities: A theory of covenants and collateral .

———. 2020. The paradox of pledgeability. *Journal of Financial Economics* 137:591–605.

Donaldson, J. R., E. R. Morrison, G. Piacentino, and X. Yu. 2020. Restructuring vs. bankruptcy. *Columbia Law and Economics Working Paper* .

Duffie, D. 2010. *Dynamic asset pricing theory.* Princeton University Press.

Fan, H., and S. M. Sundaresan. 2000. Debt valuation, renegotiation, and optimal dividend policy. *The Review of Financial Studies* 13:1057–99.

François, P., and E. Morellec. 2004. Capital structure and asset prices: Some effects of bankruptcy procedures. *The Journal of Business* 77:387–411.

Gertner, R., and D. Scharfstein. 1991. A theory of workouts and the effects of reorganization law. *The Journal of Finance* 46:1189–222.

Gilson, S. C., and M. R. Vetsuypens. 1994. Creditor control in financially distessed firms: Empirical evidence. *Wash. ULQ* 72:1005–.

Glode, V., and C. C. Opp. 2023. Private renegotiations and government interventions in credit chains. *The Review of Financial Studies* 36:4502–45.

Hackbarth, D., C. A. Hennessy, and H. E. Leland. 2007. Can the trade-off theory explain debt structure? *The Review of Financial Studies* 20:1389–428.

Hackbarth, D., and D. C. Mauer. 2012. Optimal priority structure, capital structure, and investment. *The Review of Financial Studies* 25:747–96.

Hartman-Glaser, B., S. Mayer, and K. Milbradt. 2023. A theory of asset-and cash flow-based financing. Working Paper, Technical report, National Bureau of Economic Research.

Hu, Y., F. Varas, and C. Ying. 2021. Debt maturity management. Working Paper, Working Paper.

Ivashina, V., and B. Vallee. 2020. Weak credit covenants. Working Paper, National Bureau of Economic Research.

Lambrecht, B. M. 2001. The impact of debt financing on entry and exit in a duopoly. *The Review of Financial Studies* 14:765–804.

Morellec, E. 2001. Asset liquidity, capital structure, and secured debt. *Journal of financial economics* 61:173–206.

Nini, G., D. C. Smith, and A. Sufi. 2012. Creditor control rights, corporate governance, and firm value. *The Review of Financial Studies* 25:1713–61.

Rampini, A. A., and S. Viswanathan. 2013. Collateral and capital structure. *Journal of Financial Economics* 109:466–92.

———. 2020. Collateral and secured debt. *Unpublished working paper, Duke University* .

Sundaresan, S., and N. Wang. 2007. Investment under uncertainty with strategic debt service. *American Economic Review* 97:256–61.

Zhong, H. 2021. A dynamic model of optimal creditor dispersion. *The Journal of Finance* 76:267–316.

Table 1: Parameter values

This table shows our baseline parameter values.

r	Risk-free rate	0.05
γ	Shareholder discount rate	0.1
σ	diffusion volatility	0.4
λ	Arrival rate of cashflow shocks	1.5
β	Cashflow-shock severity	4.3
θ	Profitability of capital	0.5
π	Recovery value of capital in default	0.9
ρ	Priority claims / capital	0.8
ν	Convexity of secured default risk	5.5
ϕ	Scale of secured default risk	1
ξ	Cost of investment	1.1
δ	Depreciation	0
τ	Corporate tax rate	0.21
α	Probability of liability management	0
ζ	Size of secured coalition	0.6
ε	Exchange rate in liability management	0.996
h_0	Equity issue fixed cost	0.01
h_1	equity issue proportional cost	0.01

Table 2: Model solution

We assume the parameter values listed in Table 1. This tables show the average results for 100000 simulations. We set time interval to be 0.01 and set terminal time to be $T = \min\{100, T_*\}$, where T_* is the time of default. We set $x_0 = \widetilde{x}$.

Moment	Model optimum
Leverage	0.370
Secured debt share	0.339
Secured debt credit spread	0.0013
Unsecured debt credit spread	0.0297
Credit spread difference	0.0284
Firm value for $\overline{s} = 1/2$	1.908
Firm value for $\overline{s} = 0$	1.850
equity value without debt	1.510

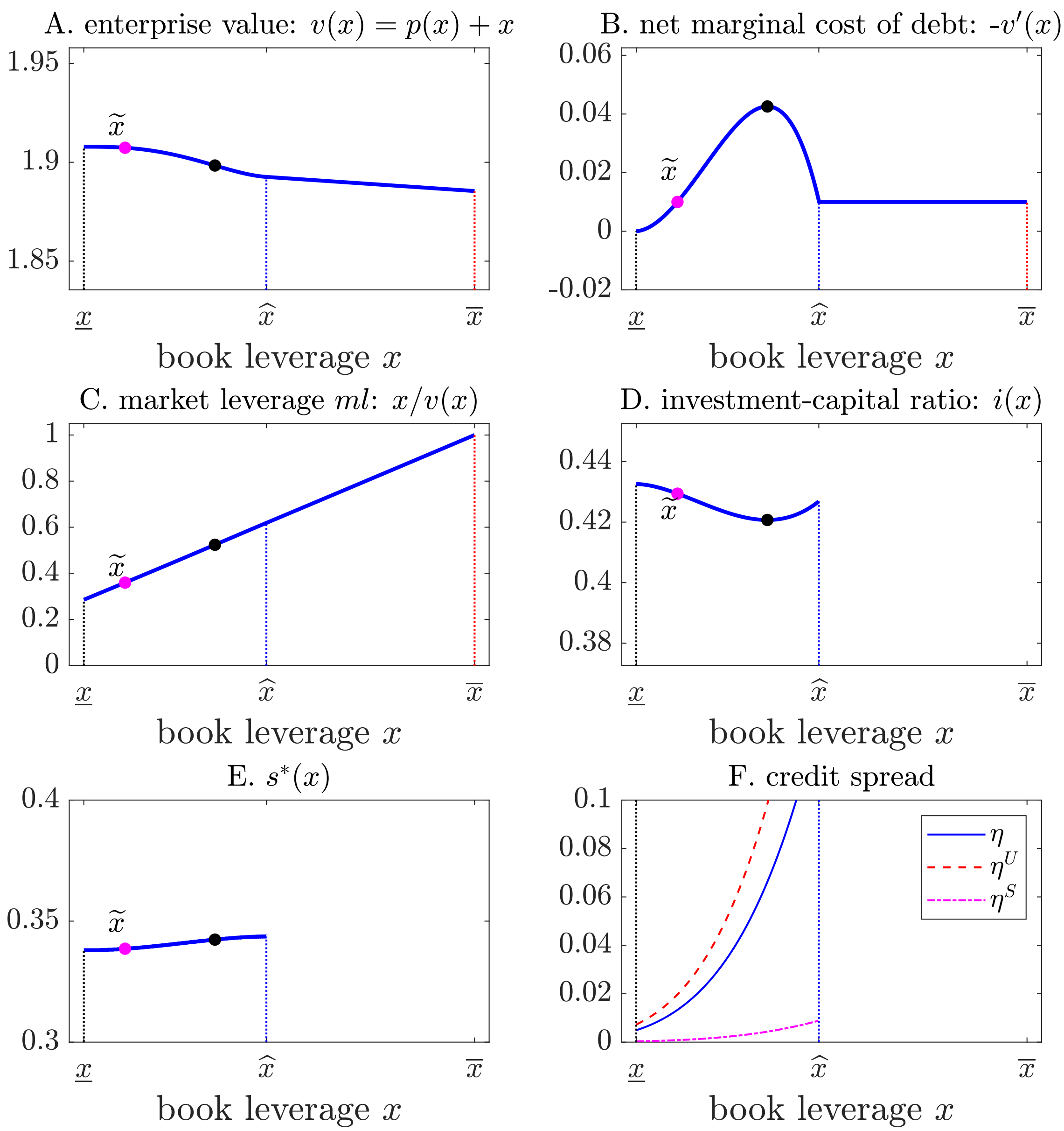

Figure 1: The parameter values are given in Table 1.