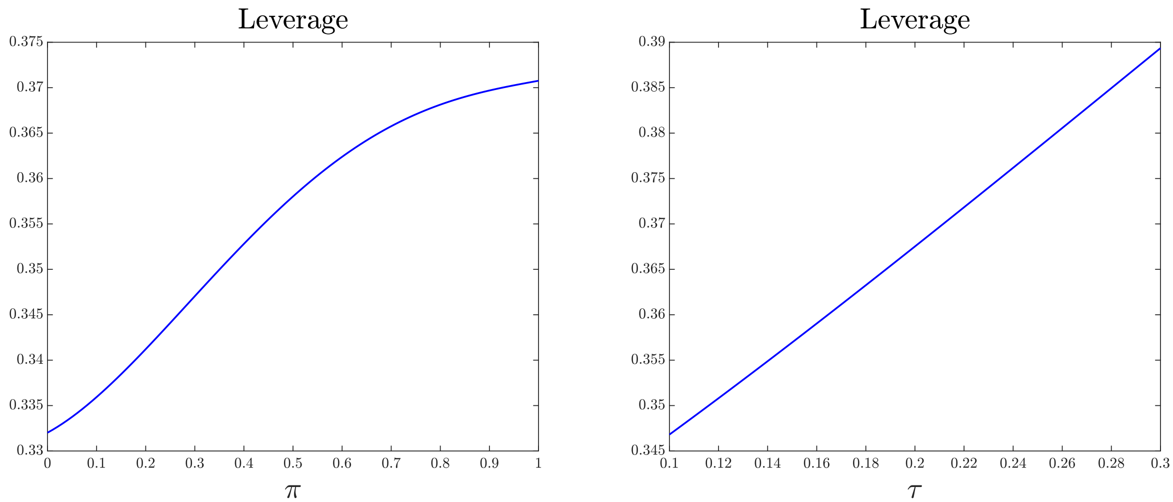

Figure 2: All parameter values are given in Table 1.

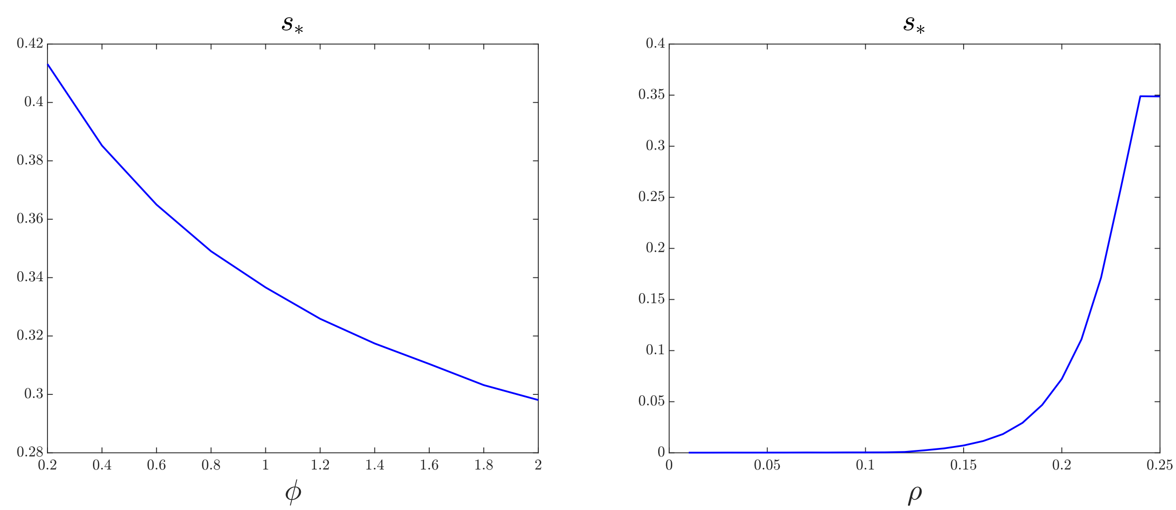

Figure 3: All parameter values are given in Table 1.

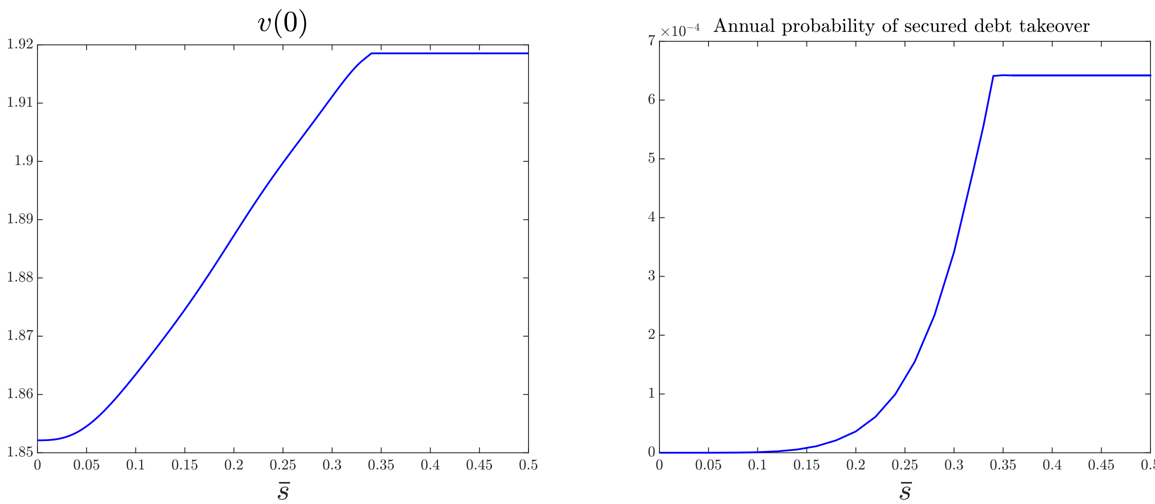

Figure 4: All parameter values are given in Table 1.

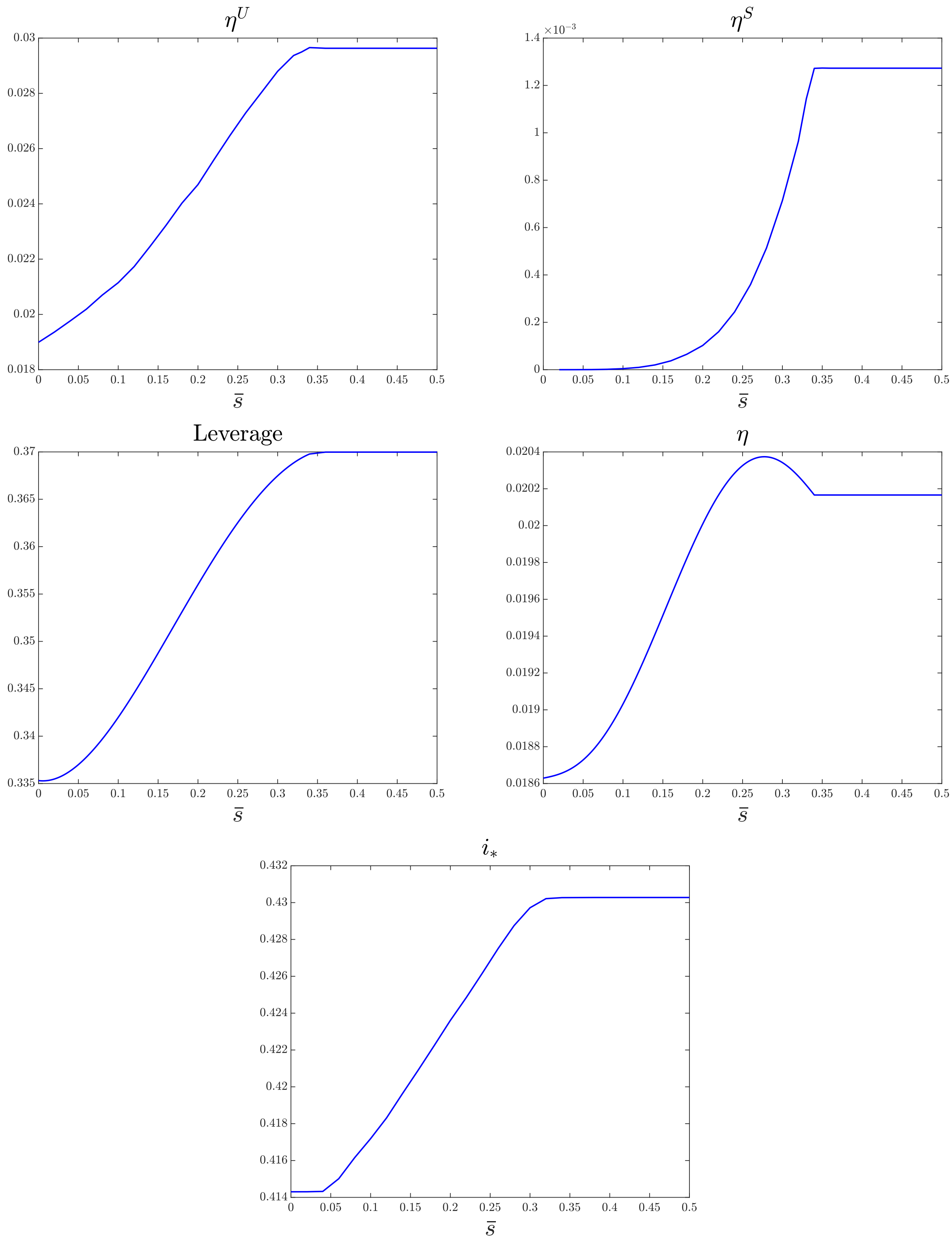

Figure 5: All parameter values are given in Table 1.

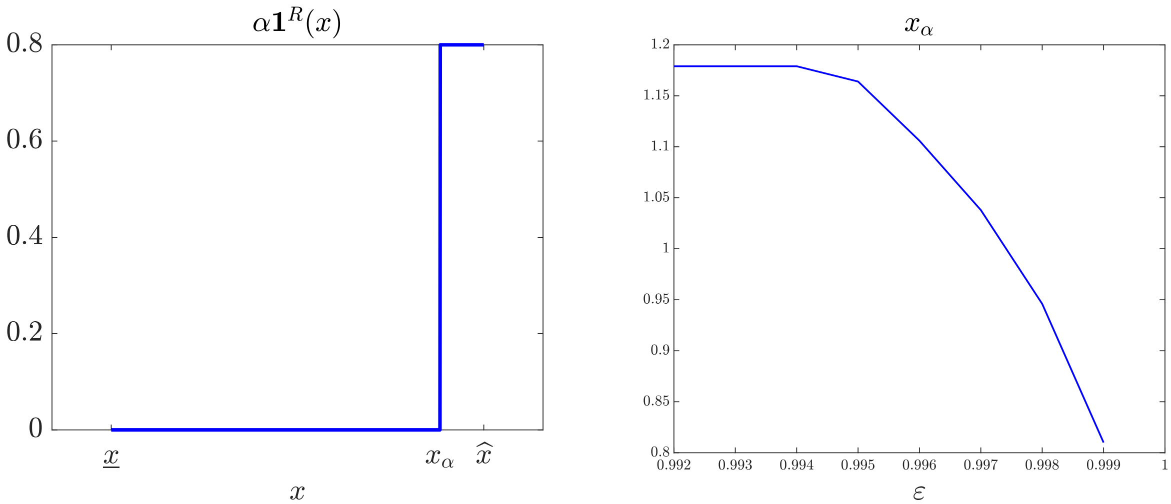

Figure 6: The parameter values are given in Table 1 but $\alpha = 0.8$.

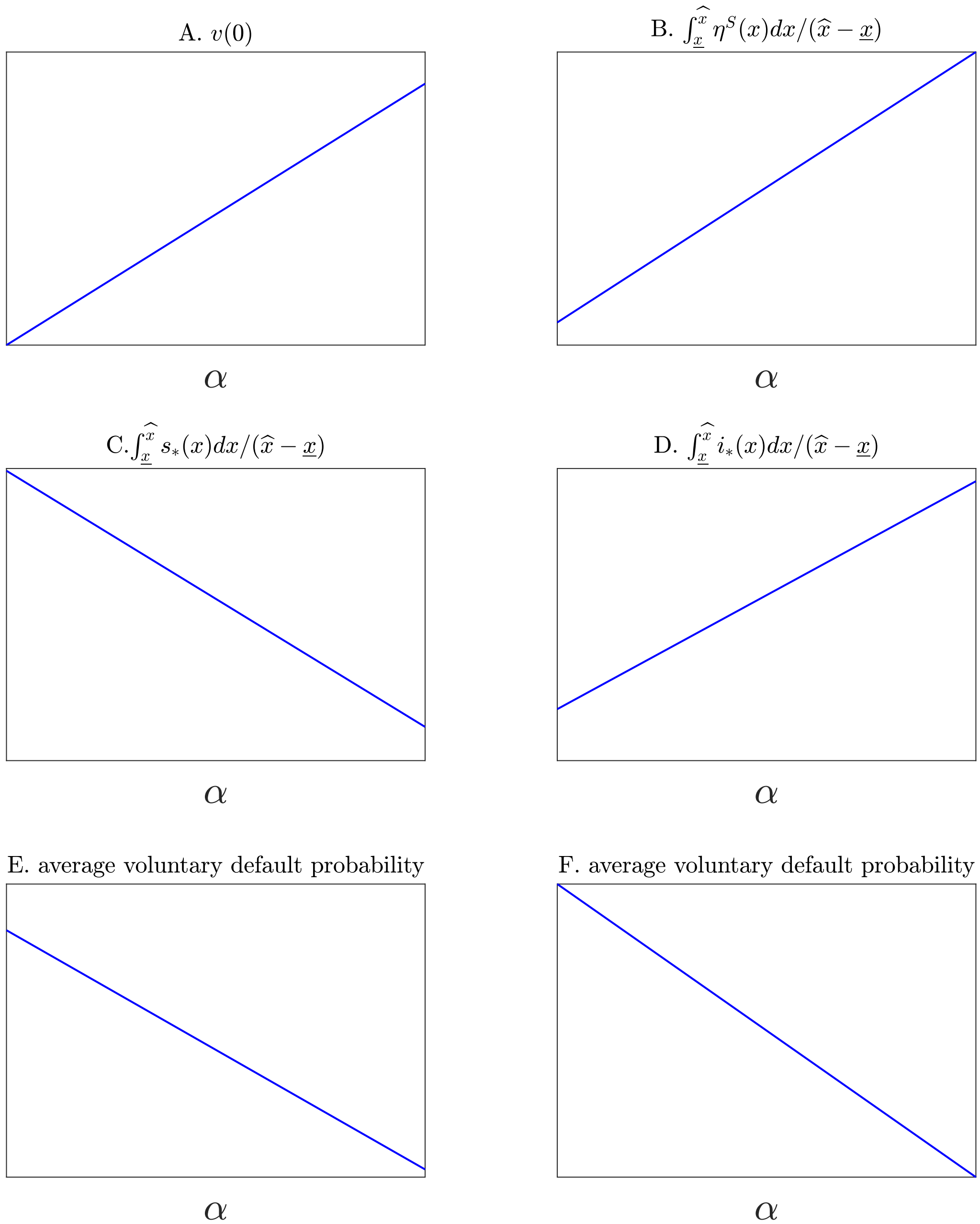

Figure 7: All parameter values are given in Table 1. The average forced default probability equals $\frac{\int_{\underline{x}}^{\widehat{x}} 1-e^{-\phi(s_*(x)x)^{\nu}}dx}{\widehat{x}-\underline{x}}$ and average voluntary default probability equals $\frac{\int_{\underline{x}}^{\widehat{x}} 1-e^{-\lambda((1-\alpha\mathbf{1}^R(x))F(Z_*(x))+\alpha\mathbf{1}^R(x)F(Z_*(x)(1-s_*(x)\zeta(1-\varepsilon))))}dx}{\widehat{x}-\underline{x}}$

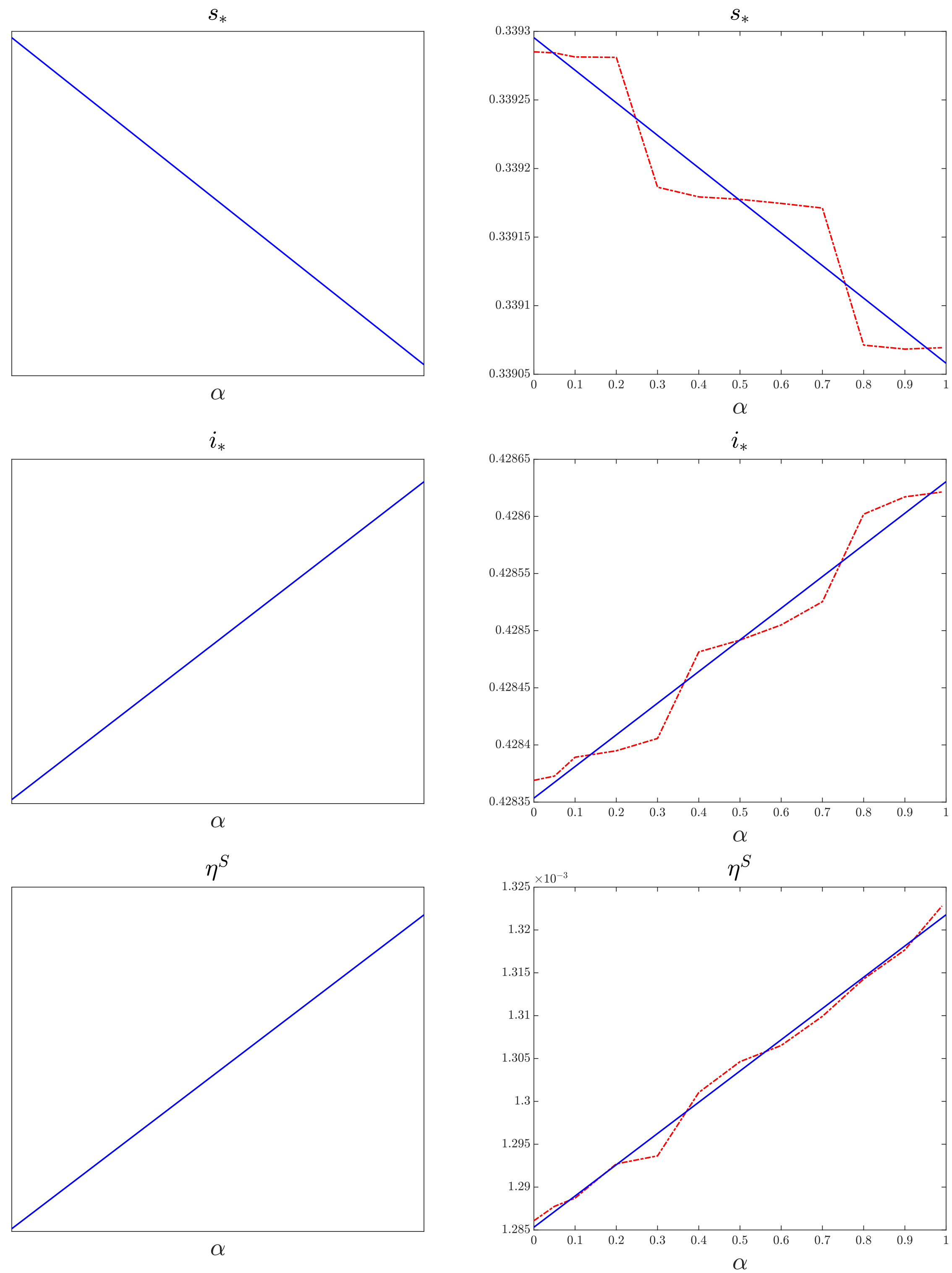

Figure 8: All parameter values are given in Table 1.

A Mathematical details

In this appendix, we derive expressions for the credit spreads η_t^S, η_t^U. We also derive the Hamilton Jacobi Bellman (HJB) equation for equity holders.

A.1 Secured credit spread

Recall that

$$s_t X_t (1 + r dt) = \mathbb{P}\left(Def_{t,t+dt} = 0 \right) s_t X_t (1 + (r + \eta_t^S) dt) + \mathbb{E}\Big[\mathcal{R}_{t+dt}^{Sec} Def_{t,t+dt} = 1 \Big]. \quad \text{(A.1)}$$

Since secured debt is fully collateralized by equation (9), secured lenders get full recovery if they force a default. In the homogeneous solution, the only state variable is the ratio:

$$x_t = \frac{X_t}{K_t}. \quad \text{(A.2)}$$

Consider the feedback strategy $s_t = s_*(x_t)$, where function $s_*(\cdot)$ is to be determined. Denote

$$p(x) = \frac{P(K, X)}{K}, \quad v(x) = \frac{V(K, X)}{K} = p(x) + x. \quad \text{(A.3)}$$

Voluntary default only occurs after a capital shock in which $Z < Z_*(x_t)$, where

$$Z_*(x) = \max\{Z \in [0, 1] : p(x/Z) = 0\}. \quad \text{(A.4)}$$

A.1.1 Scenario 1: Forced default

If secured lenders force a default, they get full recovery because we assume that $sx < \pi$ or $SX < \pi K$. This occurs with probability $\phi(sx)^{\nu} dt$.

A.1.2 Scenario 2: Endogenous default, no restructuring

If a shock occurs and there is no restructuring, equity chooses to default if $Z < Z_*(x_{t-})$. If there is a default, secured lenders recover $K_{t-} \max(s_*(x_{t-})x_{t-}, Z\pi)$. This occurs with probability $\lambda(1-\alpha)F\left(Z_*(x_{t-}) \right) dt$.

A.1.3 Scenario 3: Restructuring

We assume the following timing for a restructuring:

1. A capital shock occurs
2. Before anyone sees how bad it is (Z), the firm offers an exogenous liability management transaction: with probability α, randomly select a fraction $\zeta \in [1/2, 1]$ of secured lenders, offer an exchange rate ϵ
3. Lenders decide whether to accept, doing so reduces total debt load by $(1-\epsilon)\zeta s_{t-} X_{t-}$
4. Firm decides whether to default
5. Payoffs realize

If a shock occurs, there is a restructuring with probability α. This erases a fraction $\zeta(1-\epsilon)$ of secured debt. It reduces total debt by $(1-\epsilon)\zeta s_{t-} X_{t-}$. Note this assumes the restructuring is accepted - in the region where restructurings are rejected, we can simply set $\alpha = 0$ in the final formula.

After a shock and restructuring, the firm defaults if $Z < Z_*(x_{t-}\big[1 - s_*(x_{t-})\zeta(1-\epsilon)\big])$, which reflects the new amount of debt post restructuring. If there is no default, total value to secured lenders is $K_{t-}\big[s_*(x_{t-})x_{t-}(1-\zeta(1-\epsilon))\big]$.[17] This occurs with probability

$$\lambda\alpha\big[1 - F\left(Z_*(x_{t-}\big[1 - s_*(x_{t-})\zeta(1-\epsilon)\big]) \right)\big]dt.$$

We now consider the scenario in which the firm defaults after restructuring. A measure $\zeta s_*(x_{t-})X_{t-}$ of secured lenders have face value ϵ while a measure $(1-\zeta)s_*(x_{t-})X_{t-}$ have face value 1. It follows that the total recovery to secured lenders is

$$\min\Big(Z\pi K_{t-}, s_*(x_{t-})X_{t-}\big[\epsilon\zeta + (1-\zeta)\big]\Big). \tag{A.5}$$

This occurs with probability

$$\lambda\alpha F\left(Z_*(x_{t-}\big[1 - s_*(x_{t-})\zeta(1-\epsilon)\big]) \right) dt$$

A.1.4 Combining default scenarios

For simplicity, we drop time subscripts, with all variables assumed to be evaluated at the left limit $t-$. Define

$$Z^S(x) = \min\left(Z_*(x), \frac{sx}{\pi} \right). \tag{A.6}$$

If there is no restructuring, secured lenders get full recovery in a shock with $Z > Z^S(x)$.

[17] This is also what lenders expect to receive in this scenario before learning whether they are in the coalition. A continuum of lenders with measure $s_*(x_{t-})X_{t-}$ each have face value 1. They know with probability $1-\zeta$ they will keep face value 1. With probability ζ they will get face value ϵ. Conditional on no default and a restructuring, the expected face value is $1-\zeta+\epsilon\zeta$. Multiplying by the mass of the continuum gives the total value above.

That's because either the firm doesn't default ($Z > Z_*(x)$) or there is enough value for full repayment $Z\pi K > sX$.

Define

$$x^{res}(x,s) \equiv x(1 - s\zeta(1-\varepsilon)) \tag{A.7}$$

and

$$Z^V(x,s) = \min\left(Z_*(x^{res}(x,s)), \frac{sx(1-\zeta(1-\varepsilon))}{\pi} \right). \tag{A.8}$$

If there is a restructuring, the new face value of debt is $x^{res}(x,s)K$ and secured lenders get full recovery on their new face value $sX(1-\zeta(1-\varepsilon))$ when $Z > Z^V(x,s)$ by the same logic as above.

Piecing together the above scenarios, the breakeven condition for secured lenders, which says that the risk free return must equal the expected return for creditors to break even in expectation, is:

$$sX(1+rdt) = \underbrace{sX(1+(r+\eta^S)dt)\left(1 - [\lambda(1-\alpha)F(Z_*(x)) + \lambda\alpha]dt\right)}_{\text{Full recovery unless (shock+no restructure+default) or (shock+restructure)}} \tag{A.9}$$

$$+\underbrace{\lambda\alpha\left[1 - F\left(Z_*(x^{res}(x,s))\right)\right]dt\left[sX(1+(r+\eta^S)dt)(1-\zeta(1-\varepsilon))\right]}_{\text{Full recovery net haircut if (shock+restructure+no default)}} \tag{A.10}$$

$$+\underbrace{\lambda\alpha dt\left[\left(F(Z_*(x^{res}(x,s))) - F(Z^V(x,s))\right)(1-\zeta(1-\epsilon))sX + \pi K F(Z^V(x,s))E[Z|Z < Z^V(x,s)]\right]}_{\text{Default recovery if (shock+restructure+default)}} \tag{A.11}$$

$$+\underbrace{\lambda(1-\alpha)dt\left[\left(F(Z_*(x)) - F(Z^S(x))\right)sX + \pi K F(Z^S(x))E[Z|Z < Z^S(x)]\right]}_{\text{Default recovery if (shock + no restructure + default)}}. \tag{A.12}$$

Note scenario 1 (forced default) does not appear because secured lenders get full recovery. Dividing by $sXdt$ and taking dt to zero,

$$0 = \eta^S - [\lambda(1-\alpha)F(Z_*(x)) + \lambda\alpha] \tag{A.13}$$

$$+\lambda\alpha\big[1 - F\left(\, Z_*(x^{res}(x,s))\,\right)\big]\Big[(1-\zeta(1-\varepsilon))\Big] \tag{A.14}$$

$$+\lambda\alpha\left[\left(\, F(Z_*(x^{res}(x,s))) - F(Z^V(x,s))\,\right)(1-\zeta(1-\varepsilon)) + \frac{\pi}{sx}F(Z^V(x,s))E[Z|Z < Z^V(x,s)]\right] \tag{A.15}$$

$$+\lambda(1-\alpha)\left[\left(\, F(Z_*(x)) - F(Z^S(x))\,\right) + \frac{\pi}{sx}F(Z^S(x))E[Z|Z < Z^S(x)]\right]. \tag{A.16}$$

This provides a closed form for η^S. Specifically, recall that $F(Z) = Z^\beta$, implying that $E[Z] = \hat{b} \equiv \frac{\beta}{\beta+1}$ and $\int_a^b Z dF(Z) = \frac{\beta}{\beta+1}(b^{\beta+1} - a^{\beta+1})$. We thus have:

$$\eta_t^S = \eta^S(s_t, x_t, Z_*(x_t)), \tag{A.17}$$

where $\eta^S(\cdot,\cdot,\cdot)$ is defined as follows:

$$\begin{aligned}
\eta^S &= \lambda(1-\alpha)(Z_*(x))^\beta + \lambda\alpha \\
&\quad - \lambda\alpha\Big[\,\big(\,1-(Z^V(x,s))^\beta\,\big)\,(1-\zeta(1-\varepsilon)) + \frac{\pi\hat{b}}{sx}(Z^V(x,s))^{\beta+1}\Big] \\
&\quad - \lambda(1-\alpha)\Big[\,\big(\,(Z_*(x))^\beta - (Z^S(x))^\beta\,\big) + \frac{\pi\hat{b}}{sx}(Z^S(x))^{\beta+1}\Big] \\
&= \lambda(1-\alpha)\Big[(Z^S(x))^\beta - \frac{\pi\hat{b}}{sx}(Z^S(x))^{\beta+1}\Big] \\
&\quad - \lambda\alpha\Big[\,\big(\,1-(Z^V(x,s))^\beta\,\big)\,(1-\zeta(1-\varepsilon)) + \frac{\pi\hat{b}}{sx}(Z^V(x,s))^{\beta+1} - 1\Big]. \qquad (A.18)
\end{aligned}$$

A.1.5 Condition for secured lenders accepting offer

Finally, we consider the condition for secured lenders accepting the transaction. Recall that this is only relevant after a shock occurs. Given a shock has occurred, if the coalition of secured lenders reject, their expected recovery per dollar of face value is

$$(1-F(Z^S(x))) + F(Z^S(x))\frac{\pi}{sx}E[Z|Z < Z^S(x)]. \qquad (A.19)$$

The first term captures full recovery if secured lenders are unimpaired. The second term captures all secured lenders sharing the full recovery value.

If the coalition of secured lenders accept, the expected recovery per dollar of face value is

$$(1-F(Z_\zeta(x,s)))\epsilon + F(Z_\zeta(x,s))\frac{\pi}{sx\zeta}E[Z|Z < Z_\zeta(x,s)], \qquad (A.20)$$

where accepting secured lenders are impaired on their new face value if Z is less than

$$Z_{\zeta}(x,s) \equiv \min(Z_*(x^{res}(x,s))\,,\,\frac{\zeta\epsilon sx}{\pi}). \tag{A.21}$$

The first term above captures getting ϵ dollars per original dollar of face value if the new super secured debt is unimpaired. The second term above captures splitting the full recovery value with the original coalition, which originally had $sx\zeta$.

Putting this together, secured lenders accept a restructuring offer at (x,s) if and only if

$$(1-(Z_{\zeta}(x,s))^{\beta})\varepsilon + (Z_{\zeta}(x,s))^{\beta+1}\frac{\pi\hat{b}}{sx\zeta} > (1-(Z^S(x))^{\beta}) + (Z^S(x))^{\beta+1}\frac{\pi\hat{b}}{sx}. \tag{A.22}$$

We use this to verify that (11) holds.

A.2 Unsecured credit spread

Recall that

$$\begin{aligned} X_t(1-s_t)(1+rdt) = \mathbb{P}\left(\,Def_{t,t+dt}=0\,\right)\,X_t(1-s_t)(1+(r+\eta_t^U)dt) \\ + \mathbb{E}\Big[\mathcal{R}_{t+dt}^{Unsec}Def_{t,t+dt}=1\Big]. \end{aligned} \tag{A.23}$$

To start, we consider the case where no subordination occurs. Let

$$Z^{U,l,N}(x) = \min\Big(Z_*(x),\frac{s_*(x)x}{\pi-\rho}\Big). \tag{A.24}$$

Then $Z < Z^{U,l,N}(x_t)$ implies firm defaults in the period $[t,t+dt]$ and $Z(\pi-\rho)K_t < s(x_t)X_t$

so there is nothing left for unsecured creditors in default (assuming no subordination).

Let

$$Z^{U,h,N}(x) = \min(Z_*(x), \frac{x}{\pi - \rho}). \tag{A.25}$$

Then $Z < Z^{U,h,N}(x_t)$ implies $Z(\pi - \rho)K_t < X_t$ so unsecured debt is impaired, while $Z > Z^{U,h,N}(x_t)$ implies unsecured debt is unimpaired. It's clear that $Z^{U,h,N} \geq Z^{U,l,N}$. Then, if there is no subordination, unsecured recovery is 0 for $Z < Z^{U,l,N}(x_t)$, it is $(\pi - \rho)ZK_t - s_t X_t$ for $Z \in (Z^{U,l,N}(x_t), Z^{U,h,N}(x_t))$, and it is $(1 - s_t)X_t$ for $Z > Z^{U,h,N}(x_t)$.

Next, suppose subordination occurs. The situation is the same, except we must account for the reduced amount of secured debt and the different default threshold:

$$Z^{U,l,V}(x) = \min(Z_*(x^{res}(x, s_*(x))), \frac{(1 - \zeta + \varepsilon\zeta)s_*(x)x}{\pi - \rho}) \tag{A.26}$$

Then $Z < Z^{U,l,V}(x_t)$ implies equity defaults and unsecured get nothing.

Let

$$Z^{U,h,V}(x) = \min\left(Z_*(x^{res}(x, s_*(x))), \frac{x\left[(1 - s_*(x)) + s_*(x)(1 - \zeta + \varepsilon\zeta)\right]}{\pi - \rho}\right). \tag{A.27}$$

Unsecured lenders get full recovery if $Z > Z^{U,h,V}$ by the same logic described above. Let $f_{sub} = (1 - \zeta + \varepsilon\zeta)$.

In the following, to ease the notation, we denote $Z_*(x_t)$, $Z_*(x^{res}(x_t, s_*(x_t)))$, $Z^{U,h,N}(x_t)$, $Z^{U,l,N}(x_t)$, $Z^{U,h,V}(x_t)$ and $Z^{U,l,V}(x_t)$ as Z_*, Z_*^{res}, $Z^{U,h,N}$, $Z^{U,l,N}$, $Z^{U,h,V}$ and $Z^{U,l,V}$, respectively. We similarly drop time subscripts and evaluate at left limits $t-$. Then the unsecured breakeven condition is

$$
\begin{aligned}
&(1-s)X(1+rdt) = (1-s)X(1+(r+\eta^U)dt) \\
&\times \left(1 - \left[\lambda \left(\ (1-\alpha)F(Z_*) + \alpha F(Z_*^{res}) \ \right) + \phi(\frac{sX}{K})^\nu \right] dt \right) \\
&+ \phi(\frac{sX}{K})^\nu dt \min\left[(1-s)X, (\pi-\rho)K - sX\right]^+ \\
&+ \lambda(1-\alpha)dt \left(\left(\ F(Z_*) - F(Z^{U,h,N}) \ \right) (1-s)X \right. \\
&\left. + \mathbb{E}\left[\ \left(\ (\pi-\rho)ZK - sX \ \right) \mathbf{1} \left(\ Z \in (Z^{U,l,N}, Z^{U,h,N}) \ \right) \right] \ \right) \\
&+ \lambda\alpha dt \left(\left(\ F(Z_*^{res}) - F(Z^{U,h,V}) \ \right) (1-s)X \right. \\
&\left. + \mathbb{E}\left[\ \left(\ (\pi-\rho)ZK - f_{sub}sX \ \right) \mathbf{1} \left(\ Z \in (Z^{U,l,V}, Z^{U,h,V}) \ \right) \right] \ \right) .
\end{aligned}
\tag{A.28}
$$

Divide by $(1-s)Xdt$ and let dt go to zero:

$$
\begin{aligned}
0 = \eta^U &- \left[\lambda \left(\ (1-\alpha)F(Z_*) + \alpha F(Z_*^{res}) \ \right) + \phi(sx)^\nu \right] + \phi(sx)^\nu \min\left[1, \frac{\pi-\rho-sx}{(1-s)x}\right]^+ \\
&+ \lambda(1-\alpha) \left(\left(\ F(Z_*) - F(Z^{U,h,N}) \ \right) + \mathbb{E}\left[\frac{(\pi-\rho)Z - sx}{(1-s)x} \mathbf{1} \left(\ Z \in (Z^{U,l,N}, Z^{U,h,N}) \ \right) \right] \ \right) \\
&+ \lambda\alpha \left(\left(\ F(Z_*^{res}) - F(Z^{U,h,V}) \ \right) + \mathbb{E}\left[\frac{(\pi-\rho)Z - f_{sub}sx}{(1-s)x} \mathbf{1} \left(\ Z \in (Z^{U,l,V}, Z^{U,h,V}) \ \right) \right] \ \right) .
\end{aligned}
$$

Rearranging, this gives a closed form:

$$
\eta_t^U = \boldsymbol{\eta}^U(s_t, x_t, Z_*(x_t)), \tag{A.29}
$$

where $\boldsymbol{\eta}^U(\cdot,\cdot,\cdot,\cdot)$ is defined as follow

$$\begin{aligned}
&\boldsymbol{\eta}^U(s,x,Z_*) = \lambda(1-\alpha)(Z_*)^\beta + \phi(sx)^\nu - \phi(sx)^\nu \min\left[1, \frac{\pi-\rho-sx}{(1-s)x}\right]^+ \\
&- \lambda(1-\alpha)\Bigg((Z_*)^\beta - (Z^{U,h,N}(x))^\beta - \frac{s}{(1-s)}\big[(Z^{U,h,N}(x))^\beta - (Z^{U,l,N}(x))^\beta)\big] \\
&+ \frac{\hat{b}(\pi-\rho)}{(1-s)x}\Big((Z^{U,h,N}(x))^{\beta+1} - (Z^{U,l,N}(x))^{\beta+1}\Big) \Bigg) \\
&+ \lambda\alpha \Bigg((Z^{U,h,V}(x))^\beta + \frac{f_{sub}s}{(1-s)}\big[(Z^{U,h,V}(x))^\beta - (Z^{U,l,V}(x))^\beta)\big] \\
&- \frac{\hat{b}(\pi-\rho)}{(1-s)x}\big[(Z^{U,h,V}(x))^{\beta+1} - (Z^{U,l,V}(x))^{\beta+1}\big] \Bigg) .
\end{aligned} \tag{A.30}$$

Denote $\boldsymbol{\eta}(s,x,Z_*) = s\boldsymbol{\eta}^S(s,x,Z_*) + (1-s)\boldsymbol{\eta}^U(s,x,Z_*)$.

A.3 HJB equation with costly equity issuance

On the internal financing region, we have (1) and

$$dX_t = \Big(-\theta K_t + I_t + (1-\tau)C_t\Big)dt - \alpha(X_t/K_t)\zeta(1-\varepsilon)s_{t-}X_{t-}d\mathcal{J}_t, \tag{A.31}$$

where $\alpha(X_t/K_t) = \alpha\mathbf{1}^R(x_t)$ if the secured acceptance condition (A.22) is met and zero otherwise. Using the homogeneity property, equation (7) implies $C_t = c(x_t, s_t, Z_*(x_t))K_t$ where:

$$c(x_*, s_*, Z_*) \equiv x_* \Bigg(r + \boldsymbol{\eta}^S(x_*, s_*, Z_*)s_* + \boldsymbol{\eta}^U(x_*, s_*, Z_*)(1-s_*) \Bigg), \tag{A.32}$$

$Z_*(\cdot)$ is given by (A.4), $\boldsymbol{\eta}^S(\cdot,\cdot,\cdot)$ is given by (A.18), and $\boldsymbol{\eta}^U(\cdot,\cdot,\cdot)$ is given by (A.30).

Then we can derive the following HJB equation for the equity value function $P(K,X)$

on the internal financing region

$$\begin{aligned}\gamma P(K,X) \;=\; & \max_{I,s\in[0,\min\{1,\frac{\pi}{x}\}]} \Big(-\theta K + I + (1-\tau)c\Big(\frac{X}{K}, s, Z_*\big(\frac{X}{K}\big)\Big)K\Big)P_X(K,X) \\ & + K\Big(\psi\left(\frac{I}{K}\right) - \delta\Big)P_K(K,X) + \frac{1}{2}\sigma^2K^2P_{KK}(K,X) \\ & + \lambda\Big[-P(K,X) + (1-\alpha(X/K))\int_0^1 P(ZK,X)dF(Z) \\ & + \alpha(X/K)\int_0^1 P(ZK, X - sX\zeta(1-\varepsilon))dF(Z)\Big] \\ & + \phi\Big(\frac{sX}{K}\Big)^{\nu}\Big[(\pi K - \rho K - X)^+ - P(K,X)\Big]. \end{aligned} \tag{A.33}$$

Using $x = X/K$ and $p(x) = P(K,X)/K$, we have $P_X(K,X) = p'(x)$, $P_K(K,X) = p(x) - xp'(x)$, $KP_{KK}(K,X) = x^2p''(x)$. Substituting them with $i = I/K$ into (A.33), we derive the HJB equation for $p(x)$:[18]

$$\begin{aligned}\gamma p(x) \;=\; & \max_{i,s\in[0,\min\{1,\frac{\pi}{x}\}]} \Big(-\theta + i + (1-\tau)c(x,s,Z_*(x))\Big)p'(x) + \frac{1}{2}\sigma^2x^2p''(x) \\ & + \Big(\psi(i) - \delta\Big)\Big(p(x) - xp'(x)\Big) \\ & + \lambda\Big[(1-\alpha\mathbf{1}^R(x))\int_0^1 Zp\Big(\frac{x}{Z}\Big)dF(Z) \\ & \quad + \alpha\mathbf{1}^R(x)\int_0^1 Zp\Big(x\frac{1-s\zeta(1-\varepsilon)}{Z}\Big)dF(Z) - p(x)\Big] \\ & + \phi\big(sx\big)^{\nu}\left[(\pi-\rho-x)^+ - p(x)\right]. \end{aligned} \tag{A.34}$$

Using (A.4), we have $p(x/Z) = 0$ for $Z < Z_*(x)$ and $p(x(1-s\zeta(1-\varepsilon))/Z)$ for $Z < Z_*^{res}(x,s)$. Substituting them into (A.34), we can derive (35).

[18] Note that $P(ZK, X - sX\zeta(1-\epsilon))/ZK = p\left((X/K)\times(1-s\zeta(1-\epsilon))/Z \right)$.

A.4 Algorithm for numerical solution

A.4.1 The Case Without Restructuring

In this section, we consider the case $\alpha = 0$. Note that (A.32) is independent with Z_*^{res} when $\alpha = 0$. Then, let function $c(x_*, s_*, Z_*)$ be given by (A.32) with $\alpha = 0$.

1. According to (35), we define the operators

$$\begin{aligned}
&\mathcal{A}p^{i,s}(x) \\
=&\frac{1}{2}\sigma^2 x^2 p''(x) + \Big(-\theta + i + (1-\tau)c(x, s, Z_*(x)) - x\Big(\psi(i) - \delta\Big)\Big)p'(x) \\
&- \Big(\lambda + \gamma + \delta - \psi(i)\Big)p(x) \\
&+ \phi\big(sx\big)^{\nu}\left[(\pi - \rho - x)^+ - p(x)\right] = 0,
\end{aligned} \tag{A.35}$$

   and

$$\mathcal{B}p(x) := \int_{Z_*(x)}^{1} Zp(x/Z)dF(Z), \tag{A.36}$$

   where $Z_*(x)$ is defined by (A.4). Moreover, we set

$$\mathcal{A}p(x) = \max_{i\in\mathbb{R}, s\in[0,\min\{1,\frac{\pi}{x}\}]} \mathcal{A}^{i,s}p(x), \tag{A.37}$$

   where the optimal s is denoted as $s^*(x;p)$ and optimal i is denoted as $i^*(x;p)$.

   Then we derive from (35) that $\mathcal{A}p(x) + \lambda\mathcal{B}p(x) = 0$ in the debt financing region $x \in (\underline{x}, \widehat{x})$.

2. According to (21)-(35), we propose following variational inequality:

$$\max\{\mathcal{A}p(x) + \lambda\mathcal{B}p(x), 1 + p'(x)\} = 0, \quad x \in (x_{min}, \widehat{x}) \tag{A.38}$$

with boundary conditions:

$$p'(x_{min}) = -1, \quad p'(\widehat{x}) = -(1 + h_1), \tag{A.39}$$

where $x_{min} > 0$ is any sufficiently small number, and equity-issuance boundary $\widehat{x}$ is determined by (24), which implies

$$\mathcal{M}p(\widehat{x}) = p(\widehat{x}).$$

Here, $\mathcal{M}p(x)$ denotes the equity value after equity financing:

$$\mathcal{M}p(x) := \max_{\Delta x > 0} \Big[p(x - \Delta x) - h_0 - (1 + h_1)\Delta x\Big]. \tag{A.40}$$

3. Numerically, we can find $\widehat{x}$ by following steps:

   (i) Give a initial guess of the equity-issuance boundary, denoted by $\widehat{x}_0$.

   (ii) Solve equation (A.38) and obtain $p(x)$.

   (iii) Update $\widehat{x}$ using $\mathcal{M}p(x)$ as follow:

$$\widehat{x} = \widehat{x}_0 - \epsilon \mathbf{1}_{\mathcal{M}p(\widehat{x}_0) > p(\widehat{x}_0)} + \epsilon \mathbf{1}_{\mathcal{M}p(\widehat{x}_0) < p(\widehat{x}_0)}, \tag{A.41}$$

   where $\epsilon > 0$ is a sufficiently small number. The intuition is given as follow:

Firm has incentive to issue equity at x only when x is large enough such that $\mathcal{M}p(x) \geq p(\widehat{x}_0)$. Then we should have $\widehat{x} = \inf\{x : \mathcal{M}p(x) \geq p(x)\}$. Thus, $\mathcal{M}p(\widehat{x}_0) > p\widehat{x}_0$ means $\widehat{x} < \widehat{x}_0$ and $\mathcal{M}p(\widehat{x}_0) < p(\widehat{x}_0)$ means $\widehat{x} > \widehat{x}_0$.

(iv) We stop updating $\widehat{x}$ if $|\mathcal{M}p(\widehat{x}) - p(\widehat{x})|$ is sufficiently small.

4. The endogenous payout boundary $\underline{x} = \sup\{x \in [x_{min}, \widehat{x}] : p(x) = 1\}$.

5. In the region $x \geq \widehat{x}$, we derive from (28) and (31) that

$$p(x) = \max\{0, p(\widehat{x}) - (1 + h_1)(x - \widehat{x})\}, \quad x \geq \widehat{x}. \tag{A.42}$$

Next, we only need to solve equation (A.38) for a given $\widehat{x} > 0$. Here, we consider penalty method and Newton iteration.

1. Give a initial guess of equity value, denoted as $p_0(x)$. One simple choice is $p_0(x) = p_0(0) - x$. Thus, we only need to set a scale value $p_0(0)$. As for $p_0(0)$, one can set it as the equity value in the setting without debt.

2. We can solve (A.38) numerically by considering a sequence of functions $\{p_k(x)\}$, $k = 1, 2, \ldots$, as follows:

   (i) In the region $x \in (x_{min}, \widehat{x})$, $p_{k+1}(x)$, $k = 0, 1, \ldots$, solves the following equation

$$\mathcal{A}^{i_k(x), s_k(x)} p_{k+1}(x) + \lambda \mathcal{B} p_k(x) + \Upsilon \Big(1 + p'_{k+1}(x)\Big) \mathbf{1}_{1 + p'_k(x) \geq 0} = 0, \quad x \in (x_{min}, \widehat{x}), \tag{A.43}$$

$$p'_{k+1}(x_{min}) = -1, \quad p'_{k+1}(\widehat{x}) = -(1 + h_1), \tag{A.44}$$

where $\Upsilon > 0$ is the penalty parameter and for each k, $i_k(x) = i^*(x; p_k)$ and $s_k(x) = s^*(x; p_k)$ are the maximizer of (A.37) with p replace by p_k, the default boundary $\overline{x}_k := \inf\{x > 0 : p_k(x) = 0\}$ so $Z_*(x) = \frac{x}{\overline{x}_k}$. Moreover, $\mathcal{B}p_k(x)$ in (A.43) is given by

$$\mathcal{B}p_k(x) = \int_{\frac{x}{\overline{x}_k}}^{1} Zp_k(x/Z)dF(Z). \tag{A.45}$$

(ii) We solve ODE (A.43) using the finite difference method. We choose grid of x in the region $[x_{min}, \widehat{x}]$: $x_n = x_{min} + (n-1)\Delta x$, $n = 1, 2, \ldots, N+1$, where $\Delta x = \frac{\widehat{x} - x_{min}}{N}$. Using the upwind scheme, one can discretize (A.43) as follow:

$$\begin{aligned}
&p_{k+1}(x_n)\Big[|g_k(x_n)| + \sigma^2\frac{(x_n)^2}{(\Delta x)^2} + \gamma - \mu + \lambda + \rho(x_n s_k(x_n))^{\nu}\Big] \\
=&p_{k+1}(x_{n+1})\Big[\max\{g_k(x_n), 0\} + \sigma^2\frac{(x_n)^2}{2(\Delta x)^2}\Big] \\
+&p_{k+1}(x_{n-1})\Big[\max\{-g_k(x_n), 0\} + \sigma^2\frac{(x_n)^2}{2(\Delta x)^2}\Big] \\
+&\rho(x_n s_k(x_n))^{\nu}(L - R - x_n)^{+} + \lambda\mathcal{B}p_k(x_n) + \Upsilon\mathbf{1}_{1+p_k'(x_n)\geq 0},
\end{aligned} \tag{A.46}$$

where

$$g_k(x) = -\theta + i_k(x) + (1-\tau)c(x, s_k(x), Z_*(x)) - x\Big(\psi(i_k(x)) - \delta\Big) + \Upsilon\mathbf{1}_{1+p_k'(x)\geq 0}. \tag{A.47}$$

(iii) In the region $x \geq \widehat{x}$, we derive from (28) and (31) that

$$p_{k+1}(x) = \max\{0, p_{k+1}(\widehat{x}) - (1+h_1)(x - \widehat{x})\}, \quad x \geq \widehat{x}, \tag{A.48}$$

where $p_{k+1}(\widehat{x})$ is obtained in step (i) by solving (A.43)-(A.44).

3. Repeat step 2 until $\|p_k - p_{k+1}\|$ is sufficiently small.

A.4.2 The Case with Possible Restructuring

For the case $\alpha > 0$, we only need to change the definition of c and $\mathcal{B}p(x)$. Let function $c(x_*, s_*, Z_*)$be given by (A.32) and define $\mathcal{B}p(x)$ by

$$\mathcal{B}p(x) := (1 - \alpha \mathbf{1}^R(x)) \int_0^1 Zp(x/Z)dF(Z) + \alpha \mathbf{1}^R(x) \int_0^1 Zp(x(1 - s\zeta(1-\varepsilon))/Z)dF(Z). \tag{A.49}$$

B Illustrative model

This section presents a simple model to illustrate the intuition behind our results. We show how the potential for a liability-management transaction impacts a firm with one class of debt. We then explain how this impact will depend on a firm's use of secured debt, motivating our main model.

The illustrative model has three dates $t = 0, 1, 2$. At $t = 0$, a firm chooses how much debt to issue. At $t = 1$, the firm and its lender observe a signal about the future operations. There is then the potential for a liability-management transaction like Serta's transaction. We refer to this as a restructuring. At $t = 2$, the firm and lender observe the firm's value. The firm defaults or pays back debt, giving any residual value to shareholders.

Specifically, at $t = 0$, the firm issues fairly priced debt. Both the firm and its lender are risk neutral and have a discount rate of zero. The firm chooses the date-two repayment X that it will owe to the lender at $t = 2$. At $t = 0$, the lender gives the firm the expected value of the firm's future repayment, which takes into account the possiblity of a restructuring at

$t = 1$ or default at $t = 2$. In this sense, debt is issued at a competitively priced discount to face value, determined by rational expectations. We provide details below.

At $t = 1$, there is the potential for a restructuring. We let $\mathbf{1}_R$ denote an indicator equal to one if a restructuring offer is accepted. If the restructuring is accepted, a fraction ζ of lenders exchange each dollar of their old debt for ϵ dollars of new senior debt, where $\zeta \in [0.5, 1]$ and $\epsilon \in [0, 1]$ are exogenous parameters. In other words, if a restructuring is accepted, the total debt owed at $t = 2$ is reduced from X to $X\ \big(\ 1 - \zeta(1 - \epsilon)\ \big)$. We define

$$\tilde{X} = X\big[1 - \mathbf{1}_R\zeta(1 - \epsilon)\big] \tag{B.1}$$

as the debt owed at $t = 2$, taking into account the possibility of a restructuring at $t = 1$.

Additionally, at $t = 1$, the firm and lender learn whether the firm's operations are healthy. With probability $1-\lambda$, the firm and the lender observe that the firm's operations are healthy. In this case, everyone knows the firm's value is certain to equal $1 + \tau\tilde{X}$. We normalize the unlevered after-tax firm value to one for simplicity. The parameter $\tau > 0$ captures the tax benefits of debt per dollar of debt. This can be thought of as a reduced-form approach to modeling both the tax rate and the coupon rate.

With probability λ, the firm experiences a negative shock. In this case, the tax shield is not realized. The firm value is $Z \sim \text{Uniform}(0, 1)$, where we assume the uniform distribution for simplicity. Let $\mathbf{1}_Z$ denote an indicator for a negative shock at $t = 1$.

Finally, if the firm experiences a negative shock, it defaults if the firm value Z is less than the debt owed $\tilde{X}$. In default, the firm is only worth πZ for an exogenous parameter $\pi < 1$ that captures default costs.

Given this, the ex-ante firm value is

$$\max_{X} \ \mathbb{E}\Big[\ \big(\ 1 - \mathbf{1}_Z \ \big) \ (1 + \tau \tilde{X}) + \mathbf{1}_Z Z \ \big(\ 1 - (1-\pi)\mathbf{1}(Z < \tilde{X}) \ \big) \Big]. \tag{B.2}$$

Because debt is fairly priced and equity holders receive the debt proceeds at issuance, equity holders simply choose X at $t = 0$ to maximize (B.2).

The following proposition characterizes the impact of restructurings on ex-ante firm value.

Proposition 1. *Suppose that a restructuring occurs with probability α after a negative shock ($\mathbf{1}_Z = 1$) and with probability zero for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (B.2) *increases with the probability of restructuring α.*

Proposition 2. *Suppose that a restructuring occurs with probability zero after a negative shock ($\mathbf{1}_Z = 1$) and with probability α for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (B.2) *decreases with the probability of restructuring α.*

Intuitively, equity holders would like to realize the tax benefits of debt without risking the deadweight loss of default. Ideally, equity holders would issue a state-contingent debt contract that is cancelled in bad states of the world before default. In practice, many frictions make such a security infeasible (e.g., difficulty in verifying bad states, moral hazard, etc). However, restructurings introduce state-contingent repayment. If restructurings occur in the states where the tax-shield is valuable, this destroys value. If restructurings occur in states where default is likely, they create value ex-ante.

B.1 Secured and unsecured debt

The above results show that if restructurings are accepted in relatively good states of the world, they will harm firms ex-ante. If restructurings are only accepted in bad states of the world, they will benefit firms ex-ante.

When deciding whether to accept a restructuring, lenders trade off a lower face value with higher seniority. Higher seniority is particularly beneficial if seniority is lower to start with. For this reason, a restructuring offer aimed at unsecured creditors is more likely to succeed than a restructuring offer aimed at secured creditors. In other words, secured lenders are likely to only accept when the firm is very likely to default (bad states), while unsecured lenders are likely to accept when default is less likely (better states). Because of the legal constraints discussed in the previous section, this new form of liability-management transaction targets secured lenders. We thus expect these transactions to only be accepted by secured lenders in bad states of the world where default is close. Because of this, the above results suggest restructurings will improve ex-ante firm value. We now show this in our realistic dynamic model.

B.2 Proofs for Illustrative Model

Recall Proposition 1 states the following:

Proposition 1: *Suppose that a restructuring occurs with probability α after a negative shock ($\boldsymbol{1}_Z = 1$) and with probability zero for healthy firms ($\boldsymbol{1}_Z = 0$). Then ex-ante firm value* (B.2) *increases with the probability of restructuring α.*

Proof: Under the stated assumption, for any fixed X, firm value is

$$(1-\lambda)(1+\tau X) + \lambda(1-\alpha)\left(\int_X^1 ZdZ + \int_0^X \pi ZdZ\right) + \lambda\alpha\left(\int_{\hat{X}}^1 ZdZ + \int_0^{\hat{X}} \pi ZdZ\right), \tag{B.3}$$

where $\hat{X} \equiv X(1-\zeta(1-\epsilon))$. Evaluating integrals, this is

$$(1-\lambda)(1+\tau X)+\lambda(1-\alpha)\left(\frac{1-X^2}{2}+\frac{\pi X^2}{2}\right)+\lambda\alpha\left(\frac{1-\hat{X}^2}{2}+\frac{\pi\hat{X}^2}{2}\right). \tag{B.4}$$

Rearranging,

$$(1-\lambda)(1+\tau X)+\lambda\left(\frac{1-(1-\pi)X^2}{2}\right)+\lambda\alpha(1-\pi)\left(\frac{X^2-\hat{X}^2}{2}\right). \tag{B.5}$$

The last term is positive, so increasing α increases firm value for any chosen X, so firm value increases with α.

Recall Proposition 2 states the following:

Proposition 2: *Suppose that a restructuring occurs with probability zero after a negative shock ($\mathbf{1}_Z = 1$) and with probability α for healthy firms ($\mathbf{1}_Z = 0$). Then ex-ante firm value* (B.2) *decreases with the probability of restructuring α.*

Proof: Under the stated assumption, for any fixed X, we can apply the same steps to show that firm value is

$$(1-\lambda)(1+\tau X)-(1-\lambda)\alpha\tau(X-\hat{X})+\lambda\left(\frac{1-(1-\pi)X^2}{2}\right). \tag{B.6}$$

The second term is negative, so increasing α lowers firm value for any chosen X, so firm value decreases with α.